38

PHOTONS: LIGHT WAVES BEHAVING AS PARTICLES

38.1. **IDENTIFY:** Protons have mass and photons are massless.

(a) SET UP: For a particle with mass, $K = p^2/2m$.

EXECUTE: $p_2 = 2p_1$ means $K_2 = 4K_1$.

(b) SET UP: For a photon, $E = pc$.

EXECUTE: $p_2 = 2p_1$ means $E_2 = 2E_1$.

EVALUATE: The relation between E and p is different for particles with mass and particles without mass.

38.2. **IDENTIFY and SET UP:** $c = f\lambda$ relates frequency and wavelength and $E = hf$ relates energy and frequency for a photon. $c = 3.00\times10^8$ m/s. $1\text{ eV} = 1.60\times10^{-16}$ J.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{505\times10^{-9}\text{ m}} = 5.94\times10^{14}\text{ Hz}$

(b) $E = hf = (6.626\times10^{-34}\text{ J}\cdot\text{s})(5.94\times10^{14}\text{ Hz}) = 3.94\times10^{-19}\text{ J} = 2.46\text{ eV}$

(c) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(3.94\times10^{-19}\text{ J})}{9.5\times10^{-15}\text{ kg}}} = 9.1\text{ mm/s}$

EVALUATE: Compared to kinetic energies of common objects moving at typical speeds, the energy of a visible-light photon is extremely small.

38.3. **IDENTIFY and SET UP:** Apply $c = f\lambda$, $p = h/\lambda$ and $E = pc$.

EXECUTE: $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{5.20\times10^{-7}\text{ m}} = 5.77\times10^{14}\text{ Hz}.$

$p = \dfrac{h}{\lambda} = \dfrac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{5.20\times10^{-7}\text{ m}} = 1.28\times10^{-27}\text{ kg}\cdot\text{m/s}.$

$E = pc = (1.28\times10^{-27}\text{ kg}\cdot\text{m/s})(3.00\times10^8\text{ m/s}) = 3.84\times10^{-19}\text{ J} = 2.40\text{ eV}.$

EVALUATE: Visible-light photons have energies of a few eV.

38.4. **IDENTIFY and SET UP:** $P_{\text{av}} = \dfrac{\text{energy}}{t}$. $1\text{ eV} = 1.60\times10^{-19}$ J. For a photon, $E = hf = \dfrac{hc}{\lambda}$. $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$.

EXECUTE: **(a)** $\text{energy} = P_{\text{av}}t = (0.600\text{ W})(20.0\times10^{-3}\text{ s}) = 1.20\times10^{-2}\text{ J} = 7.5\times10^{16}\text{ eV}$

(b) $E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{652\times10^{-9}\text{ m}} = 3.05\times10^{-19}\text{ J} = 1.91\text{ eV}$

(c) The number of photons is the total energy in a pulse divided by the energy of one photon:

$\dfrac{1.20\times10^{-2}\text{ J}}{3.05\times10^{-19}\text{ J/photon}} = 3.93\times10^{16}\text{ photons}.$

EVALUATE: The number of photons in each pulse is very large.

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38.5. **IDENTIFY** and **SET UP:** $c = f\lambda$. The source emits $(0.05)(75\text{ J}) = 3.75\text{ J}$ of energy as visible light each second. $E = hf$, with $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\text{ m/s}}{600\times10^{-9}\text{ m}} = 5.00\times10^{14}\text{ Hz}$

(b) $E = hf = (6.63\times10^{-34}\text{ J}\cdot\text{s})(5.00\times10^{14}\text{ Hz}) = 3.32\times10^{-19}\text{ J}$. The number of photons emitted per second is $\dfrac{3.75\text{ J}}{3.32\times10^{-19}\text{ J/photon}} = 1.13\times10^{19}$ photons.

EVALUATE: **(c)** No. The frequency of the light depends on the energy of each photon. The number of photons emitted per second is proportional to the power output of the source.

38.6. **IDENTIFY** and **SET UP:** A photon has zero rest mass, so its energy and momentum are related by Eq. (37.40). Eq. (38.5) then relates its momentum and wavelength.

EXECUTE: **(a)** $E = pc = (8.24\times10^{-28}\text{ kg}\cdot\text{m/s})(2.998\times10^{8}\text{ m/s}) = 2.47\times10^{-19}\text{ J} =$

$(2.47\times10^{-19}\text{ J})(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 1.54\text{ eV}$

(b) $p = \dfrac{h}{\lambda}$ so $\lambda = \dfrac{h}{p} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{8.24\times10^{-28}\text{ kg}\cdot\text{m/s}} = 8.04\times10^{-7}\text{ m} = 804\text{ nm}$

EVALUATE: This wavelength is longer than visible wavelengths; it is in the infrared region of the electromagnetic spectrum. To check our result we could verify that the same E is given by Eq. (38.2), using the λ we have calculated.

38.7. **IDENTIFY** and **SET UP:** The stopping potential V_0 is related to the frequency of the light by $V_0 = \dfrac{h}{e}f - \dfrac{\phi}{e}$. The slope of V_0 versus f is h/e. The value f_{th} of f when $V_0 = 0$ is related to ϕ by $\phi = hf_{\text{th}}$.

EXECUTE: **(a)** From the graph, $f_{\text{th}} = 1.25\times10^{15}\text{ Hz}$. Therefore, with the value of h from part (b), $\phi = hf_{\text{th}} = 4.8\text{ eV}$.

(b) From the graph, the slope is $3.8\times10^{-15}\text{ V}\cdot\text{s}$.

$h = (e)(\text{slope}) = (1.60\times10^{-16}\text{ C})(3.8\times10^{-15}\text{ V}\cdot\text{s}) = 6.1\times10^{-34}\text{ J}\cdot\text{s}$

(c) No photoelectrons are produced for $f < f_{\text{th}}$.

(d) For a different metal f_{th} and ϕ are different. The slope is h/e so would be the same, but the graph would be shifted right or left so it has a different intercept with the horizontal axis.

EVALUATE: As the frequency f of the light is increased above f_{th} the energy of the photons in the light increases and more energetic photons are produced. The work function we calculated is similar to that for gold or nickel.

38.8. **IDENTIFY** and **SET UP:** $\lambda_{\text{th}} = 272\text{ nm}$. $c = f\lambda$. $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$. At the threshold frequency, f_{th}, $v_{\text{max}} \to 0$. $h = 4.136\times10^{-15}\text{ eV}\cdot\text{s}$.

EXECUTE: **(a)** $f_{\text{th}} = \dfrac{c}{\lambda_{\text{th}}} = \dfrac{3.00\times10^{8}\text{ m/s}}{272\times10^{-9}\text{ m}} = 1.10\times10^{15}\text{ Hz}$.

(b) $\phi = hf_{\text{th}} = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(1.10\times10^{15}\text{ Hz}) = 4.55\text{ eV}$.

(c) $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(1.45\times10^{15}\text{ Hz}) - 4.55\text{ eV} = 6.00\text{ eV} - 4.55\text{ eV} = 1.45\text{ eV}$

EVALUATE: The threshold wavelength depends on the work function for the surface.

38.9. **IDENTIFY** and **SET UP:** Eq. (38.3): $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi = \dfrac{hc}{\lambda} - \phi$. Take the work function ϕ from Table 38.1. Solve for v_{max}. Note that we wrote f as c/λ.

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EXECUTE: $\frac{1}{2}mv_{\text{max}}^2 = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{235\times10^{-9}\ \text{m}} - (5.1\ \text{eV})(1.602\times10^{-19}\ \text{J/1 eV})$

$\frac{1}{2}mv_{\text{max}}^2 = 8.453\times10^{-19}\ \text{J} - 8.170\times10^{-19}\ \text{J} = 2.83\times10^{-20}\ \text{J}$

$v_{\text{max}} = \sqrt{\frac{2(2.83\times10^{-20}\ \text{J})}{9.109\times10^{-31}\ \text{kg}}} = 2.49\times10^{5}\ \text{m/s}$

EVALUATE: The work function in eV was converted to joules for use in Eq. (38.3). A photon with $\lambda = 235\ \text{nm}$ has energy greater then the work function for the surface.

38.10. **IDENTIFY** and **SET UP:** $\phi = hf_{\text{th}} = \frac{hc}{\lambda_{\text{th}}}$. The minimum ϕ corresponds to the minimum λ.

EXECUTE: $\phi = \frac{hc}{\lambda_{\text{th}}} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{700\times10^{-9}\ \text{m}} = 1.77\ \text{eV}$

EVALUATE: A photon of wavelength 700 nm has energy 1.77 eV.

38.11. **IDENTIFY**: The photoelectric effect occurs. The kinetic energy of the photoelectron is the difference between the initial energy of the photon and the work function of the metal.

SET UP: $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi,\ \ E = hc/\lambda.$

EXECUTE: Use the data for the 400.0-nm light to calculate ϕ. Solving for ϕ gives $\phi = \frac{hc}{\lambda} - \frac{1}{2}mv_{\text{max}}^2 =$

$\frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{400.0\times10^{-9}\ \text{m}} - 1.10\ \text{eV} = 3.10\ \text{eV} - 1.10\ \text{eV} = 2.00\ \text{eV}.$ Then for 300.0 nm, we have $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{300.0\times10^{-9}\ \text{m}} - 2.00\ \text{eV},$ which gives

$\frac{1}{2}mv_{\text{max}}^2 = 4.14\ \text{eV} - 2.00\ \text{eV} = 2.14\ \text{eV}.$

EVALUATE: When the wavelength decreases the energy of the photons increases and the photoelectrons have a larger minimum kinetic energy.

38.12. **IDENTIFY** and **SET UP:** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$, where V_0 is the stopping potential. The stopping potential in volts equals eV_0 in electron volts. $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$ and $f = c/\lambda$.

EXECUTE: **(a)** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$ so

$eV_0 = hf - \phi = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{250\times10^{-9}\ \text{m}} - 2.3\ \text{eV} = 4.96\ \text{eV} - 2.3\ \text{eV} = 2.7\ \text{eV}.$ The stopping potential is 2.7 electron volts.

(b) $\frac{1}{2}mv_{\text{max}}^2 = 2.7\ \text{eV}$

(c) $v_{\text{max}} = \sqrt{\frac{2(2.7\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}} = 9.7\times10^{5}\ \text{m/s}$

EVALUATE: If the wavelength of the light is decreased, the maximum kinetic energy of the photoelectrons increases.

38.13. **(a) IDENTIFY:** First use Eq. (38.4) to find the work function ϕ.

SET UP: $eV_0 = hf - \phi$ so $\phi = hf - eV_0 = \frac{hc}{\lambda} - eV_0$

EXECUTE: $\phi = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{254\times10^{-9}\ \text{m}} - (1.602\times10^{-19}\ \text{C})(0.181\ \text{V})$

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$\phi = 7.821\times10^{-19}\ \text{J} - 2.900\times10^{-20}\ \text{J} = 7.531\times10^{-19}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 4.70\ \text{eV}$

IDENTIFY and **SET UP:** The threshold frequency f_{th} is the smallest frequency that still produces photoelectrons. It corresponds to $K_{max} = 0$ in Eq. (38.3), so $hf_{th} = \phi$.

EXECUTE: $f = \dfrac{c}{\lambda}$ says $\dfrac{hc}{\lambda_{th}} = \phi$

$$\lambda_{th} = \frac{hc}{\phi} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{7.531\times10^{-19}\ \text{J}} = 2.64\times10^{-7}\ \text{m} = 264\ \text{nm}$$

(b) EVALUATE: As calculated in part (a), $\phi = 4.70$ eV. This is the value given in Table 38.1 for copper.

38.14. **IDENTIFY:** The acceleration gives energy to the electrons which is then given to the x ray photons.

SET UP: $E = hc/\lambda$, so $\dfrac{hc}{\lambda} = eV$, where λ is the wavelength of the x ray and V is the accelerating voltage.

EXECUTE: $$\lambda = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(15.0\times10^{3}\ \text{V})} = 8.29\times10^{-11}\ \text{m} = 0.0829\ \text{nm}.$$

EVALUATE: This wavelength certainly is in the x ray region of the electromagnetic spectrum.

38.15. **IDENTIFY:** Apply Eq. (38.6).

SET UP: For a 4.00-keV electron, $eV_{AC} = 4000$ eV.

EXECUTE: $$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}} \Rightarrow \lambda_{min} = \frac{hc}{eV_{AC}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(4000\ \text{V})} = 3.11\times10^{-10}\ \text{m}$$

EVALUATE: This is the same answer as would be obtained if electrons of this energy were used. Electron beams are much more easily produced and accelerated than proton beams.

38.16. **IDENTIFY** and **SET UP:** $\dfrac{hc}{\lambda} = eV$, where λ is the wavelength of the x ray and V is the accelerating voltage.

EXECUTE: **(a)** $$V = \frac{hc}{e\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.150\times10^{-9}\ \text{m})} = 8.29\ \text{kV}$$

(b) $$\lambda = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(30.0\times10^{3}\ \text{V})} = 4.14\times10^{-11}\ \text{m} = 0.0414\ \text{nm}$$

EVALUATE: Shorter wavelengths require larger potential differences.

38.17. **IDENTIFY:** Energy is conserved when the x ray collides with the stationary electron.

SET UP: $E = hc/\lambda$, and energy conservation gives $\dfrac{hc}{\lambda} = \dfrac{hc}{\lambda'} + K_e$.

EXECUTE: Solving for K_e gives $K_e = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda'}\right) =$

$$(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})\left(\frac{1}{0.100\times10^{-9}\ \text{m}} - \frac{1}{0.110\times10^{-9}\ \text{m}}\right).\ K_e = 1.81\times10^{-16}\ \text{J} = 1.13\ \text{keV}.$$

EVALUATE: The electron does not get all the energy of the incident photon.

38.18. **IDENTIFY** and **SET UP:** The wavelength of the x rays produced by the tube is given by $\dfrac{hc}{\lambda} = eV$.

$\lambda' = \lambda + \dfrac{h}{mc}(1 - \cos\phi)$. $\dfrac{h}{mc} = 2.426\times10^{-12}$ m. The energy of the scattered x ray is $\dfrac{hc}{\lambda'}$.

EXECUTE: **(a)** $$\lambda = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(18.0\times10^{3}\ \text{V})} = 6.91\times10^{-11}\ \text{m} = 0.0691\ \text{nm}$$

(b) $\lambda' = \lambda + \dfrac{h}{mc}(1 - \cos\phi) = 6.91\times10^{-11}\ \text{m} + (2.426\times10^{-12}\ \text{m})(1 - \cos 45.0°)$.

$\lambda' = 6.98\times10^{-11}\ \text{m} = 0.0698\ \text{nm}$.

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(c) $E=\dfrac{hc}{\lambda'}=\dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{6.98\times10^{-11}\ \text{m}}=17.8\ \text{keV}$

EVALUATE: The incident x ray has energy 18.0 keV. In the scattering event, the photon loses energy and its wavelength increases.

38.19. **IDENTIFY:** Apply Eq. (38.7): $\lambda'-\lambda=\dfrac{h}{mc}(1-\cos\phi)=\lambda_C(1-\cos\phi)$

SET UP: Solve for $\lambda':\lambda'=\lambda+\lambda_C(1-\cos\phi)$.

The largest λ' corresponds to $\phi=180°$, so $\cos\phi=-1$.

EXECUTE: $\lambda'=\lambda+2\lambda_C=0.0665\times10^{-9}\ \text{m}+2(2.426\times10^{-12}\ \text{m})=7.135\times10^{-11}\ \text{m}=0.0714\ \text{nm}$. This wavelength occurs at a scattering angle of $\phi=180°$.

EVALUATE: The incident photon transfers some of its energy and momentum to the electron from which it scatters. Since the photon loses energy its wavelength increases, $\lambda'>\lambda$.

38.20. **IDENTIFY:** Apply Eq. (38.7): $\cos\phi=1-\dfrac{\Delta\lambda}{(h/mc)}$.

SET UP: $\dfrac{h}{mc}=0.002426\ \text{nm}$

EXECUTE: **(a)** $\Delta\lambda=0.0542\ \text{nm}-0.0500\ \text{nm}$, $\cos\phi=1-\dfrac{0.0042\ \text{nm}}{0.002426\ \text{nm}}=-0.731$, and $\phi=137°$.

(b) $\Delta\lambda=0.0521\ \text{nm}-0.0500\ \text{nm}$. $\cos\phi=1-\dfrac{0.0021\ \text{nm}}{0.002426\ \text{nm}}=0.134$. $\phi=82.3°$.

(c) $\Delta\lambda=0$, the photon is undeflected, $\cos\phi=1$ and $\phi=0$.

EVALUATE: The shift in wavelength is larger as ϕ approaches 180°. The photon loses energy in the collision, so the wavelength increases.

38.21. **IDENTIFY** and **SET UP:** The shift in wavelength of the photon is $\lambda'-\lambda=\dfrac{h}{mc}(1-\cos\phi)$ where λ' is the wavelength after the scattering and $\dfrac{h}{mc}=\lambda_C=2.426\times10^{-12}$ m. The energy of a photon of wavelength λ is $E=\dfrac{hc}{\lambda}=\dfrac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{\lambda}$. Conservation of energy applies to the collision, so the energy lost by the photon equals the energy gained by the electron.

EXECUTE: **(a)** $\lambda'-\lambda=\lambda_C(1-\cos\phi)=(2.426\times10^{-12}\ \text{m})(1-\cos35.0°)=4.39\times10^{-13}\ \text{m}=4.39\times10^{-4}\ \text{nm}$.

(b) $\lambda'=\lambda+4.39\times10^{-4}\ \text{nm}=0.04250\ \text{nm}+4.39\times10^{-4}\ \text{nm}=0.04294\ \text{nm}$.

(c) $E_\lambda=\dfrac{hc}{\lambda}=2.918\times10^{4}\ \text{eV}$ and $E_{\lambda'}=\dfrac{hc}{\lambda'}=2.888\times10^{4}\ \text{eV}$ so the photon loses 300 eV of energy.

(d) Energy conservation says the electron gains 300 eV of energy.

EVALUATE: The photon transfers energy to the electron. Since the photon loses energy, its wavelength increases.

38.22. **IDENTIFY:** The change in wavelength of the scattered photon is given by Eq. 38.7:

$$\frac{\Delta\lambda}{\lambda}=\frac{h}{mc\lambda}(1-\cos\phi)\Rightarrow\lambda=\frac{h}{mc\left(\dfrac{\Delta\lambda}{\lambda}\right)}(1-\cos\phi).$$

SET UP: For backward scattering, $\phi=180°$. Since the photon scatters from a proton, $m=1.67\times10^{-27}$ kg.

EXECUTE: $\lambda=\dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(3.00\times10^{8}\ \text{m/s})(0.100)}(1+1)=2.65\times10^{-14}\ \text{m}$.

EVALUATE: The maximum change in wavelength, h/mc, is much smaller for scattering from a proton than from an electron.

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38.23. **IDENTIFY:** During the Compton scattering, the wavelength of the x ray increases by 1.0%, which means that the x ray loses energy to the electron.

SET UP: $\Delta\lambda = \frac{h}{mc}(1-\cos\phi)$ and $\frac{h}{mc} = 2.426\times10^{-12}$ m. $\lambda' = 1.010\lambda$ so $\Delta\lambda = 0.010\lambda$.

EXECUTE: $\cos\phi = 1 - \frac{\Delta\lambda}{h/mc} = 1 - \frac{(0.010)(0.900\times10^{-10}\text{ m})}{2.426\times10^{-12}\text{ m}} = 0.629$, so $\phi = 51.0°$.

EVALUATE: The scattering angle is less than 90°, so the x ray still has some forward momentum after scattering.

38.24. **IDENTIFY:** Compton scattering occurs. We know speed, and hence the kinetic energy, of the scattered electron. Energy is conserved.

SET UP: $\frac{hc}{\lambda} = \frac{hc}{\lambda'} + E_e$ where $E_e = \frac{1}{2}mv^2$.

EXECUTE: $E_e = \frac{1}{2}mv^2 = \frac{1}{2}(9.108\times10^{-31}\text{ kg})(8.90\times10^6\text{ m/s})^2 = 3.607\times10^{-17}$ J. Using $\frac{hc}{\lambda} = \frac{hc}{\lambda'} + E_e$,

we have $\frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{0.1385\times10^{-9}\text{ m}} = 1.434\times10^{-15}$ J. Therefore,

$\frac{hc}{\lambda'} = \frac{hc}{\lambda} - E_e = 1.398\times10^{-15}$ J, which gives $\lambda' = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{1.398\times10^{-15}\text{ J}} = 0.1421$ nm.

$\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi) = 3.573\times10^{-12}$ m, so $1-\cos\phi = 1.473$, which gives $\phi = 118°$.

EVALUATE: The photon partly backscatters, but not through 180°.

38.25. **(a) IDENTIFY and SET UP:** Use Eq. (37.36) to calculate the kinetic energy K.

EXECUTE: $K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) = 0.1547mc^2$

$m = 9.109\times10^{-31}$ kg, so $K = 1.27\times10^{-14}$ J

(b) IDENTIFY and SET UP: The total energy of the particles equals the sum of the energies of the two photons. Linear momentum must also be conserved.

EXECUTE: The total energy of each electron or positron is $E = K + mc^2 = 1.1547mc^2 = 9.46\times10^{-13}$ J. The total energy of the electron and positron is converted into the total energy of the two photons. The initial momentum of the system in the lab frame is zero (since the equal-mass particles have equal speeds in opposite directions), so the final momentum must also be zero. The photons must have equal wavelengths and must be traveling in opposite directions. Equal λ means equal energy, so each photon has energy 9.46×10^{-14} J.

(c) IDENTIFY and SET UP: Use Eq. (38.2) to relate the photon energy to the photon wavelength.

EXECUTE: $E = hc/\lambda$ so $\lambda = hc/E = hc/(9.46\times10^{-14}\text{ J}) = 2.10$ pm

EVALUATE: When the particles also have kinetic energy, the energy of each photon is greater, so its wavelength is less.

38.26. **IDENTIFY:** The uncertainty principle relates the uncertainty in the duration time of the pulse and the uncertainty in its energy, which we know.

SET UP: $E = hc/\lambda$ and $\Delta E\Delta t = \hbar/2$.

EXECUTE: $E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{625\times10^{-9}\text{ m}} = 3.178\times10^{-19}$ J. The uncertainty in the energy is 1.0% of this amount, so $\Delta E = 3.178\times10^{-21}$ J. We now use the uncertainty principle. Solving $\Delta E\Delta t = \frac{\hbar}{2}$ for the time interval gives $\Delta t = \frac{\hbar}{2\Delta E} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(3.178\times10^{-19}\text{ J})} = 1.66\times10^{-16}\text{ s} = 0.166$ fs.

EVALUATE: The uncertainty in the energy limits the duration of the pulse. The more precisely we know the energy, the longer the duration must be.

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38.27. **IDENTIFY:** The wavelength of the pulse tells us the momentum of the photon. The uncertainty in the momentum is determined by the uncertainty principle.

SET UP: $p = \dfrac{h}{\lambda}$ and $\Delta x \Delta p_x = \dfrac{\hbar}{2}$.

EXECUTE: $p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{556\times10^{-9}\ \text{m}} = 1.19\times10^{-27}\ \text{kg}\cdot\text{m/s}$. The spatial length of the pulse is

$\Delta x = c\Delta t = (2.998\times10^{8}\ \text{m/s})(9.00\times10^{-15}\ \text{s}) = 2.698\times10^{-6}\ \text{m}$. The uncertainty principle gives $\Delta x \Delta p_x = \dfrac{\hbar}{2}$.

Solving for the uncertainty in the momentum, we have

$$\Delta p_x = \frac{\hbar}{2\Delta x} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(2.698\times10^{-6}\ \text{m})} = 1.96\times10^{-29}\ \text{kg}\cdot\text{m/s}.$$

EVALUATE: This is 1.6% of the average momentum.

38.28. **IDENTIFY:** We know the beam went through the slit, so the uncertainty in its vertical position is the width of the slit.

SET UP: $\Delta y \Delta p_y = \dfrac{\hbar}{2}$ and $p_x = \dfrac{h}{\lambda}$. Call the x-axis horizontal and the y-axis vertical.

EXECUTE: **(a)** Let $\Delta y = a = 6.20\times10^{-5}\ \text{m}$. Solving $\Delta y \Delta p_y = \dfrac{\hbar}{2}$ for the uncertainty in momentum gives

$$\Delta p_y = \frac{\hbar}{2\Delta y} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(6.20\times10^{-5}\ \text{m})} = 8.51\times10^{-31}\ \text{kg}\cdot\text{m/s}.$$

(b) $p_x = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{585\times10^{-9}\ \text{m}} = 1.13\times10^{-27}\ \text{kg}\cdot\text{m/s}$. $\theta = \dfrac{\Delta p_y}{p_x} = \dfrac{8.51\times10^{-31}}{1.13\times10^{-27}} = 7.53\times10^{-4}\ \text{rad}$. The width is $(2.00\ \text{m})(7.53\times10^{-4}) = 1.51\times10^{-3}\ \text{m} = 1.51\ \text{mm}$.

EVALUATE: We must be especially careful not to confuse the x- and y-components of the momentum.

38.29. **IDENTIFY** and **SET UP:** Use $c = f\lambda$ to relate frequency and wavelength and use $E = hf$ to relate photon energy and frequency.

EXECUTE: **(a)** One photon dissociates one AgBr molecule, so we need to find the energy required to dissociate a single molecule. The problem states that it requires $1.00\times10^{5}\ \text{J}$ to dissociate one mole of AgBr, and one mole contains Avogadro's number (6.02×10^{23}) of molecules, so the energy required to dissociate one AgBr is $\dfrac{1.00\times10^{5}\ \text{J/mol}}{6.02\times10^{23}\ \text{molecules/mol}} = 1.66\times10^{-19}\ \text{J/molecule}$.

The photon is to have this energy, so $E = 1.66\times10^{-19}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 1.04\ \text{eV}$.

(b) $E = \dfrac{hc}{\lambda}$ so $\lambda = \dfrac{hc}{E} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{1.66\times10^{-19}\ \text{J}} = 1.20\times10^{-6}\ \text{m} = 1200\ \text{nm}$

(c) $c = f\lambda$ so $f = \dfrac{c}{\lambda} = \dfrac{2.998\times10^{8}\ \text{m/s}}{1.20\times10^{-6}\ \text{m}} = 2.50\times10^{14}\ \text{Hz}$

(d) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(100\times10^{6}\ \text{Hz}) = 6.63\times10^{-26}\ \text{J}$

$E = 6.63\times10^{-26}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 4.14\times10^{-7}\ \text{eV}$

(e) EVALUATE: A photon with frequency $f = 100$ MHz has too little energy, by a large factor, to dissociate a AgBr molecule. The photons in the visible light from a firefly do individually have enough energy to dissociate AgBr. The huge number of 100 MHz photons can't compensate for the fact that individually they have too little energy.

38.30. **IDENTIFY:** The number N of visible photons emitted per second is the visible power divided by the energy hf of one photon.

SET UP: At a distance r from the source, the photons are evenly spread over a sphere of area $A = 4\pi r^2$.

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EXECUTE: **(a)** $N = \frac{P}{hf} = \frac{(200\text{ W})(0.10)}{h(5.00\times10^{14}\text{ Hz})} = 6.03\times10^{19}\text{ photons/sec}.$

(b) $\frac{N}{4\pi r^2} = 1.00\times10^{11}\text{ photons/sec}\cdot\text{cm}^2$ gives

$$r = \left(\frac{6.03\times10^{19}\text{photons/sec}}{4\pi(1.00\times10^{11}\text{ photons/sec}\cdot\text{cm}^2)}\right)^{1/2} = 6930\text{ cm} = 69.3\text{ m}.$$

EVALUATE: The number of photons emitted per second by an ordinary household source is very large.

38.31. **IDENTIFY** and **SET UP:** $f = \frac{c}{\lambda}$. The $(f,\ V_0)$ values are: $(8.20\times10^{14}\text{ Hz}, 1.48\text{ V})$, $(7.41\times10^{14}\text{ Hz}, 1.15\text{ V})$, $(6.88\times10^{14}\text{ Hz}, 0.93\text{ V})$, $(6.10\times10^{14}\text{ Hz}, 0.62\text{ V})$, $(5.49\times10^{14}\text{ Hz}, 0.36\text{ V})$, $(5.18\times10^{14}\text{ Hz}, 0.24\text{ V})$. The graph of V_0 versus f is given in Figure 38.31.

EXECUTE: **(a)** The threshold frequency, f_{th}, is f where $V_0 = 0$. From the graph this is $f_{th} = 4.56\times10^{14}$ Hz.

(b) $\lambda_{th} = \frac{c}{f_{th}} = \frac{3.00\times10^8\text{ m/s}}{4.56\times10^{14}\text{ Hz}} = 658\text{ nm}$

(c) $\phi = hf_{th} = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(4.56\times10^{14}\text{ Hz}) = 1.89\text{ eV}$

(d) $eV_0 = hf - \phi$ so $V_0 = \left(\frac{h}{e}\right)f - \frac{\phi}{e}$. The slope of the graph is $\frac{h}{e}$.

$$\frac{h}{e} = \left(\frac{1.48\text{ V} - 0.24\text{ V}}{8.20\times10^{14}\text{ Hz} - 5.18\times10^{14}\text{ Hz}}\right) = 4.11\times10^{-15}\text{ V/Hz}$$ and

$h = (4.11\times10^{-15}\text{ V/Hz})(1.60\times10^{-19}\text{ C}) = 6.58\times10^{-34}\text{ J}\cdot\text{s}.$

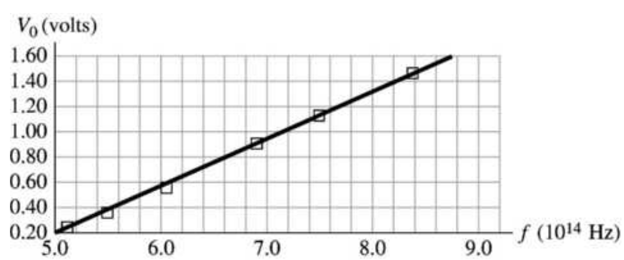

Figure 38.31

EVALUATE: The value of h from our calculation is within 1% of the accepted value.

38.32. **IDENTIFY:** The photoelectric effect occurs, so the energy of the photon is used to eject an electron, with any excess energy going into kinetic energy of the electron.

SET UP: Conservation of energy gives $hf = hc/\lambda = K_{max} + \phi$.

EXECUTE: **(a)** Using $hc/\lambda = K_{max} + \phi$, we solve for the work function:

$$\phi = hc/\lambda - K_{max} = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^8\text{ m/s})/(124\text{ nm}) - 4.16\text{ eV} = 5.85\text{ eV}$$

(b) The number N of photoelectrons per second is equal to the number of photons per second that strike the metal per second. $N\times(\text{energy of a photon}) = 2.50$ W. $N(hc/\lambda) = 2.50$ W.

$N = (2.50\text{ W})(124\text{ nm})/[(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})] = 1.56\times10^{18}\text{ electrons/s}$

(c) N is proportional to the power, so if the power is cut in half, so is N, which gives

$$N = (1.56\times10^{18}\text{ el/s})/2 = 7.80\times10^{17}\text{ el/s}$$

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(d) If we cut the wavelength by half, the energy of each photon is doubled since $E = hc/\lambda$. To maintain the same power, the number of photons must be half of what they were in part (b), so N is cut in half to 7.80×10^{17} el/s. We could also see this from part (b), where N is proportional to λ. So if the wavelength is cut in half, so is N.

EVALUATE: In part (c), reducing the power does not reduce the maximum kinetic energy of the photons; it only reduces the number of ejected electrons. In part (d), reducing the wavelength *does* change the maximum kinetic energy of the photoelectrons because we have increased the energy of each photon.

38.33. **IDENTIFY** and **SET UP:** The energy added to mass m of the blood to heat it to $T_f = 100°\text{C}$ and to vaporize it is $Q = mc(T_f - T_i) + mL_v$, with $c = 4190\ \text{J/kg}\cdot\text{K}$ and $L_v = 2.256\times10^6\ \text{J/kg}$. The energy of one photon is $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$.

EXECUTE: **(a)** $Q = (2.0\times10^{-9}\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(100°\text{C} - 33°\text{C}) + (2.0\times10^{-9}\ \text{kg})(2.256\times10^6\ \text{J/kg}) =$ $5.07\times10^{-3}\ \text{J}$. The pulse must deliver 5.07 mJ of energy.

(b) $P = \dfrac{\text{energy}}{t} = \dfrac{5.07\times10^{-3}\ \text{J}}{450\times10^{-6}\ \text{s}} = 11.3\ \text{W}$

(c) One photon has energy $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{585\times10^{-9}\ \text{m}} = 3.40\times10^{-19}\ \text{J}$. The number N of photons per pulse is the energy per pulse divided by the energy of one photon:

$$N = \frac{5.07\times10^{-3}\ \text{J}}{3.40\times10^{-19}\ \text{J/photon}} = 1.49\times10^{16}\ \text{photons.}$$

EVALUATE: The power output of the laser is small but it is focused on a small area, so the laser intensity is large.

38.34. **IDENTIFY:** The threshold wavelength λ_0 is related to the work function ϕ by $\dfrac{hc}{\lambda_0} = \phi$.

SET UP: For ϕ in eV, use $h = 4.136\times10^{-15}\ \text{eV}\cdot\text{s}$.

EXECUTE: **(a)** $\lambda_0 = \dfrac{hc}{\phi}$, and the wavelengths are: cesium: 590 nm, copper: 264 nm, potassium: 539 nm, zinc: 288 nm.

EVALUATE: **(b)** The wavelengths for copper and zinc are in the ultraviolet, and visible light is not energetic enough to overcome the threshold energy of these metals. Therefore, copper and zinc will not emit photoelectrons when irradiated with visible light.

38.35. **IDENTIFY** and **SET UP:** $\lambda' = \lambda + \dfrac{h}{mc}(1 - \cos\phi)$

$\phi = 180°$ so $\lambda' = \lambda + \dfrac{2h}{mc} = 0.09485\ \text{nm}$. Use Eq. (38.5) to calculate the momentum of the scattered photon. Apply conservation of energy to the collision to calculate the kinetic energy of the electron after the scattering. The energy of the photon is given by Eq. (38.2).

EXECUTE: **(a)** $p' = h/\lambda' = 6.99\times10^{-24}\ \text{kg}\cdot\text{m/s}$.

(b) $E = E' + E_e$; $hc/\lambda = hc/\lambda' + E_e$

$$E_e = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = (hc)\frac{\lambda' - \lambda}{\lambda\lambda'} = 1.129\times10^{-16}\ \text{J} = 705\ \text{eV}$$

EVALUATE: The energy of the incident photon is 13.8 keV, so only about 5% of its energy is transferred to the electron. This corresponds to a fractional shift in the photon's wavelength that is also 5%.

38.36. **IDENTIFY:** Compton scattering occurs. For backscattering, the scattering angle of the photon is 180°.

SET UP: Let $+x$ be in the direction of propagation of the incident photon.

$\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1 - \cos\phi)$, where $\phi = 180°$.

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EXECUTE: $\lambda' = \lambda + 2\dfrac{h}{mc} = 0.0900\times10^{-9}\ \text{m} + 4.852\times10^{-12}\ \text{m} = 0.09485\times10^{-9}\ \text{m}.$ $\dfrac{h}{\lambda} = -\dfrac{h}{\lambda'} + p_e.$ Solving for p_e gives $p_e = \dfrac{h}{\lambda} + \dfrac{h}{\lambda'} = h\left(\dfrac{\lambda+\lambda'}{\lambda\lambda'}\right) = (6.626\times10^{-34}\ \text{J}\cdot\text{s})\dfrac{9.000\times10^{-11}\ \text{m} + 9.485\times10^{-11}\ \text{m}}{(9.000\times10^{-11}\ \text{m})(9.485\times10^{-11}\ \text{m})}.$

$p_e = 1.43\times10^{-23}\ \text{kg}\cdot\text{m/s}.$

EVALUATE: The electron gains the most amount of momentum when backscattering occurs.

38.37. IDENTIFY: Compton scattering occurs, and we know the angle of scattering and the initial wavelength (and hence momentum) of the incident photon.

SET UP: $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi)$ and $p = h/\lambda.$ Let $+x$ be the direction of propagation of the incident photon and let the scattered photon be moving at $30.0°$ clockwise from the $+y$ axis.

EXECUTE: $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi) = 0.1050\times10^{-9}\ \text{m} + (2.426\times10^{-12}\ \text{m})(1-\cos 60.0°) = 0.1062\times10^{-9}\ \text{m}.$

$P_{ix} = P_{fx}.$ $\dfrac{h}{\lambda} = \dfrac{h}{\lambda'}\cos 60.0° + p_{ex}.$

$p_{ex} = \dfrac{h}{\lambda} - \dfrac{h}{2\lambda'} = h\dfrac{2\lambda' - \lambda}{(2\lambda')(\lambda)} = (6.626\times10^{-34}\ \text{J}\cdot\text{s})\dfrac{2.1243\times10^{-10}\ \text{m} - 1.050\times10^{-10}\ \text{m}}{(2.1243\times10^{-10}\ \text{m})(1.050\times10^{-10}\ \text{m})}.$

$p_{ex} = 3.191\times10^{-24}\ \text{kg}\cdot\text{m/s}.$ $P_{iy} = P_{fy}.$ $0 = \dfrac{h}{\lambda'}\sin 60.0° + p_{ey}.$

$p_{ey} = -\dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})\sin 60.0°}{0.1062\times10^{-9}\ \text{m}} = -5.403\times10^{-24}\ \text{kg}\cdot\text{m/s}.$ $p_e = \sqrt{p_{ex}^2 + p_{ey}^2} = 6.28\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

$\tan\theta = \dfrac{p_{ey}}{p_{ex}} = \dfrac{-5.403}{3.191}$ and $\theta = -59.4°.$

EVALUATE: The electron gets only part of the momentum of the incident photon.

38.38. IDENTIFY and **SET UP:** Electrical power is $VI.$ $Q = mc\Delta T.$

EXECUTE: **(a)** $(0.010)VI = (0.010)(18.0\times10^3\ \text{V})(60.0\times10^{-3}\ \text{A}) = 10.8\ \text{W} = 10.8\ \text{J/s}$

(b) The energy in the electron beam that isn't converted to x rays stays in the target and appears as thermal energy. For $t = 1.00\ \text{s},$ $Q = (0.990)VI(1.00\ \text{s}) = 1.07\times10^3\ \text{J}$ and

$\Delta T = \dfrac{Q}{mc} = \dfrac{1.07\times10^3\ \text{J}}{(0.250\ \text{kg})(130\ \text{J/kg}\cdot\text{K})} = 32.9\ \text{K}.$ The temperature rises at a rate of 32.9 K/s.

EVALUATE: The target must be made of a material that has a high melting point.

38.39. IDENTIFY and **SET UP:** Find the average change in wavelength for one scattering and use that in $\Delta\lambda$ in Eq. (38.7) to calculate the average scattering angle $\phi.$

EXECUTE: **(a)** The wavelength of a 1 MeV photon is

$$\lambda = \frac{hc}{E} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{1\times10^6\ \text{eV}} = 1\times10^{-12}\ \text{m}$$

The total change in wavelength therefore is $500\times10^{-9}\ \text{m} - 1\times10^{-12}\ \text{m} = 500\times10^{-9}\ \text{m}.$

If this shift is produced in 10^{26} Compton scattering events, the wavelength shift in each scattering event is

$$\Delta\lambda = \frac{500\times10^{-9}\ \text{m}}{1\times10^{26}} = 5\times10^{-33}\ \text{m}.$$

(b) Use this $\Delta\lambda$ in $\Delta\lambda = \dfrac{h}{mc}(1-\cos\phi)$ and solve for $\phi.$ We anticipate that ϕ will be very small, since $\Delta\lambda$ is much less than $h/mc,$ so we can use $\cos\phi \approx 1 - \phi^2/2.$

$$\Delta\lambda = \frac{h}{mc}(1-(1-\phi^2/2)) = \frac{h}{2mc}\phi^2$$

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$$\phi = \sqrt{\frac{2\Delta\lambda}{(h/mc)}} = \sqrt{\frac{2(5\times10^{-33}\text{ m})}{2.426\times10^{-12}\text{ m}}} = 6.4\times10^{-11}\text{ rad} = (4\times10^{-9})°$$

ϕ in radians is much less than 1 so the approximation we used is valid.

(c) IDENTIFY and **SET UP:** We know the total transit time and the total number of scatterings, so we can calculate the average time between scatterings.

EXECUTE: The total time to travel from the core to the surface is $(10^6\text{ y})(3.156\times10^7\text{ s/y}) = 3.2\times10^{13}\text{ s}$.

There are 10^{26} scatterings during this time, so the average time between scatterings is

$$t = \frac{3.2\times10^{13}\text{ s}}{10^{26}} = 3.2\times10^{-13}\text{ s}.$$

The distance light travels in this time is $d = ct = (3.0\times10^8\text{ m/s})(3.2\times10^{-13}\text{ s}) = 0.1\text{ mm}$

EVALUATE: The photons are on the average scattered through a very small angle in each scattering event. The average distance a photon travels between scatterings is very small.

38.40. **IDENTIFY:** Apply Eq. (38.7) to each scattering.

SET UP: $\cos(\theta/2) = \sqrt{\dfrac{1+\cos\theta}{2}}$, so $\cos\theta = 2\cos^2(\theta/2) - 1$

EXECUTE: **(a)** $\Delta\lambda_1 = (h/mc)(1-\cos\theta_1), \Delta\lambda_2 = (h/mc)(1-\cos\theta_2)$, and so the overall wavelength shift is $\Delta\lambda = (h/mc)(2 - \cos\theta_1 - \cos\theta_2)$.

(b) For a single scattering through angle $\theta, \Delta\lambda_s = (h/mc)(1-\cos\theta)$. For two successive scatterings through an angle of $\theta/2$ for each scattering,

$$\Delta\lambda_t = 2(h/mc)(1-\cos\theta/2).$$

$$1-\cos\theta = 2(1-\cos^2(\theta/2)) \text{ and } \Delta\lambda_s = (h/mc)2(1-\cos^2(\theta/2))$$

$$\cos(\theta/2) \le 1 \text{ so } 1-\cos^2(\theta/2) \ge (1-\cos(\theta/2)) \text{ and } \Delta\lambda_s \ge \Delta\lambda_t$$

Equality holds only when $\theta = 180°$.

(c) $(h/mc)2(1-\cos 30.0°) = 0.268(h/mc)$.

(d) $(h/mc)(1-\cos 60°) = 0.500(h/mc)$, which is indeed greater than the shift found in part (c).

EVALUATE: When θ is small, $1-\cos\theta \approx \theta$ and $1-\cos(\theta/2) \approx \theta/2$. In this limit $\Delta\lambda_s$ and $\Delta\lambda_t$ are approximately equal.

38.41. **(a) IDENTIFY** and **SET UP:** Conservation of energy applied to the collision gives $E_\lambda = E_{\lambda'} + E_e$, where E_e is the kinetic energy of the electron after the collision and E_λ and $E_{\lambda'}$ are the energies of the photon before and after the collision. The energy of a photon is related to its wavelength according to Eq. (38.2).

EXECUTE: $E_e = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda'}\right) = hc\left(\dfrac{\lambda' - \lambda}{\lambda\lambda'}\right)$

$$E_e = (6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})\left(\frac{0.0032\times10^{-9}\text{ m}}{(0.1100\times10^{-9}\text{ m})(0.1132\times10^{-9}\text{ m})}\right)$$

$$E_e = 5.105\times10^{-17}\text{ J} = 319\text{ eV}$$

$$E_e = \frac{1}{2}mv^2 \text{ so } v = \sqrt{\frac{2E_e}{m}} = \sqrt{\frac{2(5.105\times10^{-17}\text{ J})}{9.109\times10^{-31}\text{ kg}}} = 1.06\times10^7\text{ m/s}$$

(b) The wavelength λ of a photon with energy E_e is given by $E_e = hc/\lambda$ so

$$\lambda = \frac{hc}{E_e} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{5.105\times10^{-17}\text{ J}} = 3.89\text{ nm}$$

EVALUATE: Only a small portion of the incident photon's energy is transferred to the struck electron; this is why the wavelength calculated in part (b) is much larger than the wavelength of the incident photon in the Compton scattering.

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38.42. **IDENTIFY:** Eq. (38.7) relates λ and λ' to ϕ. Apply conservation of energy to obtain an expression that relates λ and v to λ'.

SET UP: The kinetic energy of the electron is $K=(\gamma-1)mc^2$. The energy of a photon is $E=\dfrac{hc}{\lambda}$.

EXECUTE: **(a)** The final energy of the photon is $E'=\dfrac{hc}{\lambda'}$, and $E=E'+K$, where K is the kinetic energy of the electron after the collision. Then,

$$\lambda=\frac{hc}{E'+K}=\frac{hc}{(hc/\lambda')+K}=\frac{hc}{(hc/\lambda')+(\gamma-1)mc^2}=\frac{\lambda'}{1+\dfrac{\lambda' mc}{h}\left[\dfrac{1}{(1-v^2/c^2)^{1/2}}-1\right]}.$$

($K=mc^2(\gamma-1)$ since the relativistic expression must be used for three-figure accuracy).

(b) $\phi=\arccos[1-\Delta\lambda/(h/mc)]$.

(c) $\gamma-1=\dfrac{1}{\left(1-\left(\frac{1.80}{3.00}\right)^2\right)^{1/2}}-1=1.25-1=0.250,\ \dfrac{h}{mc}=2.43\times10^{-12}\text{ m}$

$$\Rightarrow\lambda=\frac{5.10\times10^{-3}\text{ mm}}{1+\dfrac{(5.10\times10^{-12}\text{ m})(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})(0.250)}{(6.63\times10^{-34}\text{ J}\cdot\text{s})}}=3.34\times10^{-3}\text{ nm}.$$

$$\phi=\arccos\left(1-\frac{(5.10\times10^{-12}\text{ m}-3.34\times10^{-12}\text{ m})}{2.43\times10^{-12}\text{ m}}\right)=74.0°.$$

EVALUATE: For this final electron speed, $v/c=0.600$ and $K=\frac{1}{2}mv^2$ is not accurate.

38.43. **IDENTIFY:** Apply the Compton scattering formula $\lambda'-\lambda=\Delta\lambda=\dfrac{h}{mc}(1-\cos\phi)=\lambda_C(1-\cos\phi)$

(a) SET UP: Largest $\Delta\lambda$ is for $\phi=180°$.

EXECUTE: For $\phi=180°$, $\Delta\lambda=2\lambda_C=2(2.426\text{ pm})=4.85\text{ pm}$.

(b) SET UP: $\lambda'-\lambda=\lambda_C(1-\cos\phi)$

Wavelength doubles implies $\lambda'=2\lambda$ so $\lambda'-\lambda=\lambda$. Thus $\lambda=\lambda_C(1-\cos\phi)$. λ is related to E by Eq. (38.2).

EXECUTE: $E=hc/\lambda$, so smallest energy photon means largest wavelength photon, so $\phi=180°$ and $\lambda=2\lambda_C=4.85\text{ pm}$. Then

$$E=\frac{hc}{\lambda}=\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{4.85\times10^{-12}\text{ m}}=4.096\times10^{-14}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J})=0.256\text{ MeV}.$$

EVALUATE: Any photon Compton scattered at $\phi=180°$ has a wavelength increase of $2\lambda_C=4.85$ pm. 4.85 pm is near the short-wavelength end of the range of x-ray wavelengths.

38.44. **IDENTIFY:** Follow the derivation of Eq. (38.7). Apply conservation of energy and conservation of momentum to the collision.

SET UP: Use the coordinate direction specified in the problem.

EXECUTE: Momentum: $\vec{p}+\vec{P}=\vec{p}'+\vec{P}'\Rightarrow p-P=-p'-P'\Rightarrow p'=P-(p+P')$

energy: $pc+E=p'c+E'=p'c+\sqrt{(P'c)^2+(mc^2)^2}$

$\Rightarrow(pc-p'c+E)^2=(P'c)^2+(mc^2)^2=(Pc)^2+((p+p')c)^2-2P(p+p')c^2+(mc^2)^2.$

$(pc-p'c)^2+E^2=E^2+(pc+p'c)^2-2(Pc^2)(p+p')+2Ec(p-p')-4pp'c^2+2Ec(p-p')$

$$+2(Pc^2)(p+p')=0$$

$$\Rightarrow p'(Pc^2-2pc^2-Ec)=p(-Ec-Pc^2)$$

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$$\Rightarrow p' = p\frac{Ec + Pc^2}{2pc^2 + Ec - Pc^2} = p\frac{E + Pc}{2pc + (E - Pc)}$$

$$\Rightarrow \lambda' = \lambda\left(\frac{2hc/\lambda + (E - Pc)}{E + Pc}\right) = \lambda\left(\frac{E - Pc}{E + Pc}\right) + \frac{2hc}{E + Pc}$$

$$\Rightarrow \lambda' = \frac{\lambda(E - Pc) + 2hc}{E + Pc}$$

$$\text{If } E \gg mc^2,\ Pc = \sqrt{E^2 - (mc^2)^2} = E\sqrt{1 - \left(\frac{mc^2}{E}\right)^2} \approx E\left(1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 + \cdots\right)$$

$$\Rightarrow E - Pc \approx \frac{1}{2}\frac{(mc^2)^2}{E} \Rightarrow \lambda_1 \approx \frac{\lambda(mc^2)^2}{2E(2E)} + \frac{hc}{E} = \frac{hc}{E}\left(1 + \frac{m^2c^4\lambda}{4hcE}\right)$$

(b) If $\lambda = 10.6\times10^{-6}$ m, $E = 1.00\times10^{10}$ eV $= 1.60\times10^{-9}$ J

$$\Rightarrow \lambda' \approx \frac{hc}{1.60\times10^{-9}\ \text{J}}\left(1 + \frac{(9.11\times10^{-31}\ \text{kg})^2 c^4 (10.6\times10^{-6}\ \text{m})}{4hc\ (1.6\times10^{-9}\ \text{J})}\right) = (1.24\times10^{-16}\ \text{m})(1 + 56.0) = 7.08\times10^{-15}\ \text{m}.$$

(c) These photons are gamma rays. We have taken infrared radiation and converted it into gamma rays! Perhaps useful in nuclear medicine, nuclear spectroscopy, or high energy physics: wherever controlled gamma ray sources might be useful.

EVALUATE: The photon has gained energy from the initial kinetic energy of the electron. Since the photon gains energy, its wavelength decreases.

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